Lesson Plan

# Blue Moon Over the Mogollons

## Summary

This Western-set adventure introduces the reader to two distinct mathematical concepts: the probability theory associated with poker and other card games; and the physics (kinematics) determining the trajectory of a long-range projectiles. The following questions will help students to expand their awareness of both topics and are motivated by the real-world applications they will have just read in the story.

## Exercises

### Shuffling the Deck

Ma is no stranger to the card table, and her experience has taught her that seven shuffles is enough to fully mix a 52-card deck – any less, and patterns from the previous round might still be present, but no more are necessary. Can we explain this with mathematics?

a. Let's imagine the simplest possible shuffling method, the "cut". On each shuffle, the deck is split into two stacks at a random location – the bottom stack is then put on top of the top stack. Consider: how many possible split locations are possible? This is the number of possible "states" the deck can be in after this first shuffle, each with equal probability.
b. How many states are possible after two cuts? After three cuts? After N cuts?
c. To decide whether a deck is truly randomized, experts make the requirement that all possible orders of the 52 cards are (approximately) equally probable. How many such orders are there? You will need a calculator!

   Hint: Consider first how many cards could be in the bottom position. Then, out of the remaining cards, how many cards could be in the second position – and on and on until you use all the cards! You will see a pattern, and this pattern is called a "factorial".

d. Here's the big question – how many cuts do you need so that the answers to part b) and part c) are roughly the same number? This is the number of cuts which will shuffle the deck!
e. You'll notice that the needed number of cuts is far more than 7. This is because the "cut" method is not very efficient – can you think of a shuffling method which has more

possible outcomes? How many outcomes does each shuffle need to have, so that 7 shuffles is enough to achieve all possible deck orderings (the answer to part c)?

## The Eötvös Effect

Casey's rapid calculations under pressure are the only thing distinguishing the Enfield rifle from an expensive scrap of metal. This mathematics is made possible by a deep understanding of mechanical motion which physicists have painstakingly gained over centuries. We will follow in their footsteps, and study the Eötvös effect which impacts all long-range projectiles on Earth.

a. If you swing an open bucket of water in a full circle over your head, with sufficient speed, the water remains safely inside. Without using any mathematics, can you explain why? This may be a good opportunity to discuss with a classmate or two. What forces are acting on the water in the bucket throughout its motion?
b. The phenomenon you have discussed is known as centrifugal acceleration. For a bucket swinging at speed **v** and radius **r**, the centrifugal acceleration **a** = **v^2 / r**. Given a radius of 1 m, and the knowledge that the acceleration due to gravity at the surface of the earth is **g = 9.8 m/s^2**, can you calculate how fast the bucket should be swung so that it successfully overcomes gravity at the top of the circle? Since you know the radius of the circle, you can also calculate how many swings per second this represents.
c. Now, you will calculate the centrifugal acceleration of a person at the equator of the Earth. From a reliable source, look up the radius of our planet and the rotation speed at its surface. Your value for the acceleration should be small – otherwise we would go flying off the surface of the world as it spins!
d. If Casey fires her weapon due East with speed **w** (the same direction as the rotation of the surface), the centrifugal acceleration of the projectile increases to **a** = **(v+w)^2 / r**. For a typical rifle speed of 500 m/s, calculate the new centrifugal acceleration – what is the perceived gravitational acceleration for the projectile? What if the weapon were fired due West? North/South? *This is called the Eötvös effect, named after the Hungarian physicist who discovered it.*
e. Again working at the surface of the Earth calculate, how fast a rifle bullet would need to travel (Westward) for its centrifugal acceleration to exactly cancel that from gravity. Roughly speaking, this is the "escape velocity" necessary for an object to traverse from the Earth's surface to outer space.

# Solutions

## Shuffling the Deck

a. There are 51 split locations – between every adjacent pair of cards. Some students may find this difficult to work out, in which case they should be asked to answer the question for smaller "decks" until they see the pattern. This is also an opportunity to discuss whether all outcomes are really equally likely. A typical personal will make the "cut" somewhere towards the middle of the deck, effectively decreasing the number of outcomes.

b. After two cuts, 51*51 = 2601 possibilities; after three cuts, 51^3 = 132,651, and after N cuts, 51^N. A precocious student may realize that the true number is slightly smaller, as for example two different first cuts could lead to the same shuffled deck after the second cut. However they should be led to understand that this number is small, and may be neglected. This is also a useful opportunity to discuss order-of-magnitude estimates, and scientific notation, as the results will become unwieldy above N=5 or so.

c. Following the hint, the solution is 52*51*50 ... *3*2*1 = 52! = 8.066 * 10^67. Again, it will be essential for students to work in scientific notation. It may be worth mentioning that the number of stars in the observable universe is approximately 10^22, and the total number of neutrons and protons in the Earth is only 10^51.

d. This question is easy if a student has previously seen logarithms: from 51^N = 8.066 * 10^67, the solution is N = log_51(8.066 * 10^67) = 39.77, or approximately 40. However, students without logarithm experience can work it out by trial and error, experimenting with 51^N for different N until they find that 40 gives the nearest result to 8.066 * 10^67.

e. Again, many students will proceed by trial and error, looking for a number x such that x^7 = 8.066 * 10^67. However, with knowledge of exponents, the solution may be calculated directly: the seventh root of (8.066 * 10^67), or (8.066 * 10^67)^(1/7) = about 5 trillion, 5 * 10^9. Students will think they have the wrong answer, and that this is an impossible number of outcomes to achieve with just a single shuffle of a deck. It is not! The well-known riffle shuffle commonly seen at a poker table (and used by Ma) has about 4.5 * 10^15 possible outcomes, although they are not all equally likely.

## The Eötvös Effect

a. Students should be able to reason that gravity is always pulling the water, and the bucket, downward. They should also come to the conclusion that some other force must be counteracting the force of gravity to prevent the water from falling out, and that this force must point upwards at the top of the circle. However they will likely disagree about the origins of this force, and should be encouraged to think about what would happen if the bucket were simply upside down and moving at some fast horizontal speed. The origin of the centrifugal force is the inertia of an object when its container changes direction.

b. Doing the algebra gives **v^2 = r * g**, and so **v = 3.13 m/s**. Students should notice that for a larger radius, or a stronger gravitational acceleration, the necessary speed would increase. Given that the circumference of the circle is **2*pi m**, each swing will take about 2.01 s. Therefore approximately half a swing per second is required! Some students may be interested to notice that g is very similar to pi^2, and it should be emphasized that this is a total coincidence – the value of g would be different on any planet.

c. The radius of the Earth is 6,371 km. The rotation speed at the equator is about 463 m/s, which a student could also calculate using the radius and the knowledge that one rotation takes about 24 hours. As a result, a = 0.034 m/s^2, which students should intuit is much smaller than 9.8 m/s^2. It is easy to make a mistake with the km and m units here! Students should be encouraged to consider how this effect would change at other latitudes than the equator, specifically that the rotation speed decreases and so does the centrifugal acceleration.

d. Applying the formula gives a = 0.146 m/s^2, approximately four times the previous answer. This should make sense for a student if they notice that the speed of the projectile is similar to the rotation speed of the earth – the velocity has doubled, and so the acceleration should quadruple. The perceived acceleration due to gravity is then 9.8 - 0.146 = 9.654 m/s^2. In the Eastward direction, a = (463 - 500)^2 / 6371000 = 0.0002 m/s^2. In this case the firing of the weapon has caused it to nearly counterbalance the rotation of the earth. If the weapon were fired to the North or South, the Eötvös Effect would not apply – although the Coriolis effect would, which an advanced student could be encouraged to research.

e. The same logic from part b) may be applied: **v^2 = r * g**, though r is now the radius of the Earth, and the result is 7901 m/s or about 8 km/s. The true escape velocity of Earth, taking into account full Newtonian gravity, is about 11 km/s, and an advanced student might be encouraged to use conservation of energy alongside Newton's gravitational formula to derive this number.

## Story Questions

What is the conflict in “Blue Moon Over the Mogollons?? What is the other, deeper, beneath-the-surface conflict?

What is the turning point in this story (plot)?

What is the moment of truth – when we find out what is really happening (theme)?

How does the author use the Mogollons and the blue moon in this story? How do these two natural elements modify or enrich the human drama? Can you name another story in which nature plays a part (“The Lord of the Rings,” for instance) ?

Casey is similar to which member of the Avengers? How? How is she different?

What did the men in the gun-repair clinic think of Casey? Why?

Should Johnny have gone ahead and shot the lawyer? Why or why not?

What was the biggest surprise in “Blue Moon Over the Mogollons?